

***Students' Misconceptions and Mistakes in General
Mathematics Course Among Al-Quds Open
University Students***

Basem M. Shalash

تاريخ القبول 2019/7/15

تاريخ الاستلام 2019/6/3

Abstract

The aim of this study was to identify the misconceptions and mistakes made by students of a general mathematics course by analyzing the students' responses to the test questions. The study test consisted of five open-ended problems and interviews with some students. The sample of study consisted of 50 students enrolled in a general mathematics course at Al-Quds Open University. To analyze data, the descriptive analysis method was used. The results showed that some students have some misconceptions and misunderstandings in solving inequalities and operations on problem solving with fractional linear inequalities. The least ratio of common errors was (24%), which occurred when carrying out elementary algebraic operations, but the most common mistakes ratio was (46%), that occurred when students were asked to solve fractional linear inequalities.

Keywords: Inequality, common errors, misconception, general mathematics.

Introduction

Nowadays, mathematics becomes one of the most important sciences as it reveals hidden patterns that help us understand the world around us (Tonazzini et al, 2019). Now, much more than solving arithmetic and geometrical problems, today's mathematics is a diverse system that deals with data, measurements and observations from science, conclusion, deduction, and proof; and with mathematical models of natural phenomena, of human behavior, and of social systems.

Students construct their thoughts about natural phenomena to make sense of their daily life experiences that often differs from scientific views that are referred to as misconceptions or alternative concepts (Eyebiokin, 2016).

Misconceptions are one of the most important obstacles in learning mathematics. Therefore, teachers' prior knowledge is an essential for new learning, because it can help or hinder the process of understanding. Misconceptions affect understanding new concepts and information due to several reasons including lack of previous knowledge about the concept or inability of the learner to link what exists with what is new, or misinterpret the new concepts to match the previous knowledge, and then learner adheres to his previous perceptions (Won. S., 2019).

Bezuidenhout (2001) obtained that some difficulties and misconceptions in mathematics were a result of teaching approach that emphasizes a too large extended procedural aspects of the calculus and neglects a solid grounding in the underpinnings of calculus. Generally, misconceptions manifest through errors: An error can be a mistake, blunder, miscalculation or misjudge and such category falls under unsystematic errors. The challenging issue concerning misconceptions is that many people have difficulty in relinquishing misconceptions because the false concepts may be deeply ingrained in the mental map of an individual. Some people do not like

to be proven wrong and will continue clinging to a misconception in the face of evidence to the contrary. This view is consistent with that of Hammer (1996) who thought students' misconceptions: 1- are strongly held, stable cognitive structures; 2. differ from expert understanding; 3. affect in a fundamental sense how students understand natural phenomena and scientific explanations; and 4. must be overcome, avoided, or eliminated for students to achieve expert understanding (p. 99).

One of the main methods used to analyze students' errors is to classify them into certain categorizations based on an analysis of students' behaviours. Through using a cognitive information processing model and considering the specialties of mathematics, Radatz (1979) classified the errors in terms of (1) language difficulties. Mathematics is like a "foreign language" for students who need to know and understand mathematical concepts, symbols, and vocabulary. Misunderstanding the semantics of mathematics language may cause students' errors at the beginning of problem solving; (2) difficulties in processing iconic and visual representation of mathematical knowledge; (3) deficiency in requisite skills, facts, and concepts; for example, students may forget or be unable to recall related information in solving problems; (4) incorrect associations or rigidity; that is, negative transfer caused by decoding and encoding information; and (5) application of irrelevant rules or strategies. Orton (1983) classified errors into three categories as follows: (1) Structural error: is an error which arises from some failure to appreciate the relationship involved in the problem or to grasp some principle essential to solution. (2) Arbitrary error: is that error in which the subject behaved arbitrarily and failed to take into account the constraints laid down in what was given. (3) Executive error: is that error where student fails to carry out manipulations, though the principles involved may have been understood. Mathematical knowledge is interrelated and misconceptions in one branch of mathematics maybe carried into other areas of mathematics, (Kula and Bukova 2014).

Despite the agreement among researchers that algebra is an important and vital part of mathematics and learning, many studies have indicated that there are difficulties in learning algebra. Being an essential part of mathematics, Algebra is an extension of operations performed by the students on integers and fractions performed on symbols and algebraic concepts. However, fraction misconceptions greatly influence students' acquisition of algebra knowledge. Many learners find algebra complex and difficult to understand. They know that algebra has to do with variables; however, they do not understand what the letters indicate, or why they are used (Reed, 2010)

Li (2006) emphasized that students have misconceptions about inequalities that represent a difficult step up from equation for many students. The procedure of solving inequalities is deceptively similar to those of solving equations, but there are some differences that student often overlook. Teachers can help students recognize these differences and solve the inequalities by multiple solution method. Many student do not see the need to adhere to the order of operations rules and resort to solving the expression from left to right. Furthermore, many students fail to realize that parenthesis can be used to both group together as well as signal multiplication (i.e. $(20-7) = -13$). (Gardella, 2008).

Wang (2015) also described the students' strategies when dealing with inequality. Students will try to communicate what they already know about how to solve equation with solving inequalities; however, they do not connect the correct previous knowledge to the new concepts. Students tend to apply equation-solving strategies when solving inequalities. This misconception is understandable because equations and inequalities "look" similar in this study. Ciltas and Tatar (2011) stated the difficulties and misconception the students experienced in the subject of inequalities as follows:

1. Students dealing with inequalities in the same way they treat equations.

For example: $2x-1 < 0 \Rightarrow 2x-1 = 0$

2. Students forget the reversing the inequality sign when dividing/multiplying both sides of the inequality by a negative number.
For example: $-2x < 6 \Rightarrow x < -3$
3. General errors when conducting algebraic operations such as forgetting to distribute the number behind the parenthesis, or neglecting the presence of the parenthesis. For example: $3(x-1) \geq 2 \Rightarrow 3x-1 \geq 2$.

Definitions

- Mathematical inequality: an algebraic expression that contains one of the symbols ($>$, $<$, less than or greater than).
- Variable solution: Find the value (or values) of the variable (x) that make inequality statement true.
- Mathematical error in solving the inequalities: is the procedure carried out by the student in solving the inequality and violating the scientific rules for the solving procedures, that is related to concepts, generalizations, skills, or the errors made by students due to negligence or quickness and lack of attention.
- Common error in the solution: Some studies indicate that the common error is the error that is repeated in the students' answers.
- Misconception is a view or opinion that is incorrect because it is based on faulty thinking or understanding.

Questions of Study

The study test consisted of the following questions:

1. Find the solution set for the inequality $\frac{2x + 9}{3} \geq 1$. In this question, the students are expected to find the solution for the inequality distinguishing the sign " \geq ", from the equal sign.

2. Find the solution set for the inequality $\frac{5}{2-x} \geq 1$. In this question, the students are expected to find the solution for the inequality, paying more attention to the sign of $2-x$.
3. Find the solution set for the linear fractional inequality $\frac{5x+8}{10x} \geq 1$. In this question, the students are expected to find the solution by considering case that occurred due to sign of the denominator.
4. Find the solution set for the linear fractional inequality $\frac{4x}{8} + \frac{x}{2} \leq \frac{1}{5}$. In this question, the students were unable to unify the denominators for the fraction.
5. Find the solution set for the linear fractional inequality $\frac{3x}{6x+4} \geq 2$. In this question, the students were unable to memorize that the fraction cannot be distributed through the denominator.

The following two open-ended interview questions focused on :

1. How to solve linear Inequalities?

2. How to solve linear Inequalities With Fractions?

Methodology

Research Goal

The aim of this study was to identify the misconceptions and mistakes made by students of a general mathematics course by analyzing the students' responses to the study questions.

The two main symbols used in inequalities, the less than sign ($<$) and the more than sign ($>$) were given to test the students (ability) in comparing between two integers which are positive and negative. The students were also expected to represent the inequalities on a number line. The sub-topic under solving linear inequality had the highest ideal percentage (75%) among the

three sub-topics. This is because the main learning objectives focused on the sub-topic of solving linear inequality. The items given in this sub-topic were about solving different types of inequalities and finding greatest or smallest possible values from given inequalities.

Sample

The sample of the study consisted of 50 students enrolled in a general mathematics course in the first semester of the academic year 2018/2019. These students were selected randomly from Al-Quds Open University- Salfit branch.

Data Collection and Procedure

The study tool was applied to an exploratory sample outside the study population, in order to ascertain clarity of the paragraphs and their relevance when applied to the sample of the current study. The exploratory sample consisted of 23 students who enrolled in the course at the summer semester 2017/2018. The exploratory research reveals that students look positively toward the questions of study. The current study sample of students, which has already been taught how to solve inequalities by using algebra and geometric methods, was given a test of 5 open-ended questions. Semi-structured interviews have been made with 7 students in order to detect the students' errors in the mathematical learning process in detail. The purpose of this approach has been explained to each student at the beginning of the interviews. The duration of these interviews was about 65 minutes for all students.

Findings and Discussion

After grading the test, some common misconception appeared in distinct ratios.

Table 1 shows the frequencies and percentages of the answers, common errors and misconceptions among students.

Table 1 Frequencies and percentages of the answers , common errors and misconceptions among students.

Misconception	Number of Students	Percentage
Dealing with inequalities in the same way		
that they treat equations	15	30%
Students forget reversing the inequality		
Sign when dividing/multiplying both sides	22	44%
of the inequality by a negative number.		
General errors when conducting algebraic		
operations such as "cancelling everything in sight"	12	24%
Misunderstanding of unifying the denominators		
for the fraction in rational linear inequality	19	38%
As a single unit		
Misunderstanding of distributing through the		
numerator in fractional linear inequalities.		
Inability of students to memorize that the fraction	23	46%
cannot be distributed through the denominator		

Table 1 shows that 70% of the students gave the correct answer to the first question. In other words, 30 % of the students were not able to solve the inequality correctly. One of the incorrect solutions was given in Sample 1.

Sample 1. *The solution given by one of the students who failed to give the correct answer to the first question:*

$$\begin{aligned} \frac{2x+9}{3} &\geq 1 \\ 2x+9 &= 3 \\ 2x &= 3-9 = -6 \\ x &= -3 \end{aligned}$$

According to this result, we might conclude that a considerable number of students failed to answer this question correctly. As given in the Sample 2 the students obtained the equalities $2x+9=3$ and $x=-3$ by multiplying by 3 both sides of the inequality $\frac{2x+9}{3} \geq 1$.

Since the students did not understand the difference between equality and inequality, they gave incorrect answer.

Sample 2. *The solution given by one of the students who failed to give the correct answer to the second question.*

$$\begin{aligned} \frac{5}{2-x} &\geq 1 \\ 5 &\geq 2-x \\ x &\geq -3 \\ x &\in [-3, \infty) \end{aligned}$$

The second question was given a correct answer by 56 % of the students. According to this result, we might conclude that a considerable number of students failed to answer this question correctly. As given in the Sample 2., the students obtained the inequalities $5 \geq 2-x$ and $x \geq -3$ by multiplying by $2-x$ both sides of $\frac{5}{2-x} \geq 1$.

Since the students solved the inequality without taking in consideration the sign of the term $2-x$, they could not find the correct answer.

Sample 3. *The solution given by one of the students who failed to give the correct answer to the second question.*

Another mistake done in this second question was to find incorrect solution set by using the inequality $3+x \geq 0$. This inequality was found by multiplying both sides of the inequality $\frac{3+x}{2-x} \geq 0$ by $2-x$:

$$\begin{aligned} \frac{5}{2-x} &\geq 1 \\ \frac{5}{2-x} - 1 &\geq 0 \\ \frac{5-2+x}{2-x} &\geq 0 \\ \frac{3+x}{2-x} &\geq 0 \Rightarrow 3+x \geq 0 \\ x &\geq -3, [-3, \infty) \end{aligned}$$

Sample 4. *The solution given by one of the students who failed to give the correct answer to the third question.*

In the third question, nearly 76% of the students found the correct answer. In other words, 24% of the students were not able to perform operations that require problem solving with fractional linear inequalities and they made fatal error such as "cancelling everything in sight":

$$\begin{aligned} \frac{5x+8}{10x} &\geq 1 \\ \frac{5x+8}{10x} &\geq 1 \Leftrightarrow \frac{5x'+8}{10x'} \geq 1 \\ \frac{13}{10} &\geq 1 \end{aligned}$$

Sample 5. *The solution given by one of the students who failed to give the correct answer to the fourth question.*

In the fourth question, nearly 62% of the students found the correct answer. In other words, 38% of the students misunderstood unifying the denominators for the fraction in rational linear inequality such as

$$\frac{4x}{8} + \frac{x}{2} \leq \frac{1}{5}$$

$$\frac{5x}{10} \leq \frac{1}{5}$$

$$5x \leq 2$$

$$x \leq \frac{2}{5}$$

$$\left[-\infty, \frac{2}{5}\right]$$

Sample 6. *The solution given by one of the students who failed to give the correct answer to the fifth question.*

In the fifth question, nearly 54% of the students found the correct answer. In other words, 46% of the students misunderstood the distributing through the numerator in fractional linear inequalities such as:

$$\frac{3x}{6x+4} \geq 2$$

$$\frac{3x}{6x} + \frac{3x}{4} \geq 2$$

$$\frac{1}{2} + \frac{3x}{4} \geq 2$$

$$\frac{3x}{4} \geq \frac{3}{2}$$

$$\frac{x}{4} \geq \frac{1}{2} \Leftrightarrow x \geq 2$$

$$[2, \infty)$$

Interview Procedure

The interviews were made with *Student A* and *Student B* who failed to give the correct answer to the first and fifth questions, respectively.

Interviewer: Can you tell me how you solved the problem “What are the x values that satisfy the inequality $\frac{2x + 9}{3} \geq 1$?”

Student A : I think it was easy to solve the inequality like equation $\frac{2x + 9}{3} = 1$.

I found the solution as $x = -3$.

Interviewer: So, have you checked the correctness of the solution that you found for this problem?

Student A : I did not check since I was sure of it.

Interviewer: Can you check it now?

Student A: Let us write -3 in place of x ; it verifies.

Interviewer: Have you understood why the result you found is not true?

Student A: No.

Interviewer: What was a method you used in the other problem?

Student A: I solved it similarly because it was of the same logic.

Interviewer: Can you tell me how you solved *the fourth question* in the test?

Student A: First, I added the numerators and denominators separately,

$\frac{4x + x}{8 + 2} \leq \frac{1}{5}$. Then, I equated one side to another such as $\frac{5x}{10} = \frac{1}{5}$, and I

solved it, $x = \frac{2}{5}$

Interviewer: You did the same in the 5th problem. So, are you saying that you add the numerators and denominators separately?

Student A: Yes, we learned so.

Interviewer: So, have you checked the result that you found?

Student A: yes, but it does not satisfy the problem.

Interviewer: You said that the solution set doesn't satisfy the given problem. What will you say about this?

Student A: I do not know. I found it so and I wrote it.

Interviewer: Can you tell me how you solved the question “ Find the solution set for the linear fractional inequality $\frac{3x}{6x + 4} \geq 2$ ” ?

Student B: Firstly, I multiplied both sides of inequality by $6x + 4$; I collected the terms of the variable x and arrived at the solution $x \leq \frac{-8}{9}$.

Interviewer: But, what about the sign of $6x + 4$?

Student B: What? I do not know.

Interviewer: Here you have two cases due to the sign of $6x + 4$, so you have another case ?

Student B: I think that we learned so.

4. Discussion and Conclusion

In this paper we analyzed some errors and misconceptions made by undergraduate students in general mathematics. Results from tests showed that some students had a weak background of elementary mathematics such as a low level at linear inequality concepts. Performance in the study test on basic concepts emphasized the same fact, and some errors on linear inequality were observed again in fractional inequality. Learners also seem to have difficulties with algebraic operations. Some errors made by students refers to whether the inequality symbol is the greater than (symbol $>$) or the less than (symbol $<$) (Frempong, 2012). Frempong pointed out that solving linear inequalities are “similar to the techniques for solving linear equations, except that an inequality is divided or multiplied by a negative number, the sense of the inequality must be reversed” (p. 121). Li (2006), assured that student errors are the symptom of misunderstanding. Among many different types of errors, systematic errors were made by many students over a long time period and it is relatively easy

and thus possible to research with current knowledge and tools. The cause of systematic errors may relate to student's procedure knowledge, conceptual knowledge, or links between these two types of knowledge. Based on the study results the author may suggest the following recommendations:

- Faculty members should concentrate in classroom on the basic concepts and skills associated with the concept of equations and inequalities and methods of solving
- Further studies should be conducted in the field of the common errors analysis of the mathematics students about solving inequalities of two variables.
- Providing the Ministry of Education with this study and using its results to overcome the difficulties and misconceptions faced by school pupils.

References

[1] Bezuidenhout, J.(2001) Limits and continuity : some conceptions of first year students. *International Journal of Mathematics Education in Science and Technology* 32(4), 487-500.

[2] Ciltas A, Tatar E. (2011). Diagnosing learning difficulties related to the equation and inequality that contain terms with absolute value, *International Online Journal of Educational Sciences*, 2011, 3(2), 461-473.

[3] Frempong, A. A. (2012). Final exam review: intermediate algebra. NY: Finalexamsreview.com.

[4] GARDELLA F. J. (2008). INTRODUCING DIFFICULT MATHEMATICS TOPICS IN THE ELEMENTARY CLASSROOM: A TEACHER'S GUIDE TO INITIAL LESSONS, DOI: 10.4324/9780203891179.

[5] Haji M., Jamilah Y., Masitah S and Mar A. Exploring students' understanding, on 'inequalities', *Mediterranean Journal of Social Sciences* · September 2015 DOI: 10.5901/mjss.2015.v6n5s1p218.

- [6] Hammer, D. (1996). Misconceptions or P-primes: How alternative perspectives of cognitive structure influence instructional perceptions and intentions? *The Journal of the learning science* 5, 97-127.
- [7] Eyebiokin K. (2016). Influence of misconceptions and alternative conceptions On senior secondary school students' in physics, M. Thesis. https://www.academia.edu/38149438/Influence_of_Misconceptions_and_Alternative_Conceptions_on_Physics_Students_Kharis_2016.pdf
- [8] Kula, S., & Bukova Güzel, E. (2014). Misconceptions emerging in mathematics student teachers' limit instruction and their reflections. *Quality & Quantity*, 48, 3355-3372.
- [9] Li, X.(2006). Cognitive Analysis of students' errors and misconceptions in variables, equations and functions. *Journal for Research in Mathematics Education*.
- [10] Radatz, H. (1979) Error analysis in mathematics education: *Journal for Research in Mathematics Education*,10,163-172.
- [11] Reed, R. A. (2010). From learner algebraic misconceptions to reflective educator: Three cycles of an action research project, retrieved march 10, 2019, from https://researchspace.ukzn.ac.za/xmlui/bitstream/handle/10413/6073/Reed_Rosanthia_2010.pdf?sequence=1&isAllowed=y
- [12] Rowntree, R. V. Students' understandings and misconceptions of algebraic inequalities, *School Science and Mathematics*, v109 n6 p311-312 Oct 2009
- [13] Tonazzini A. , Salerno E., Abdel-Salam Z. et al.(2019), Analytical and mathematical methods for revealing hidden details in ancient manuscripts and paintings: A review, *Journal of Advanced Research*, <https://doi.org/10.1016/j.jare.2019.01.003>

[14] Wang, X. (2015) The literature review of algebra learning: focusing on the contributions to students' difficulties. *Creative Education*, 6, 144-153. <http://dx.doi.org/10.1007/BF01273663>

[15] Won S.Y. How does students' prior knowledge affect their learning? *Retrieved* March 10, 2019, from https://www.colorado.edu/ftcp/sites/default/files/attached-files/ftcp_memo_to_faculty_88_0.pdf, Colorado University, USA.